Schedule:
- Test Prep Today!
- Finish Welcome Quiz by tonight
- HW 1 closes Tuesday

2.2: Separable Differential Equations

Entry Task: *(Motivation)*

Implicitly differentiate $x^2 + y^3 = 8$ and solve for $\frac{dy}{dx}$.

Idea: Separate… integrate both sides.

Entry Task continued:

Find the *explicit* solution for

$$\frac{dy}{dx} = \frac{-2x}{3y^2}$$

with $y(0) = 2$.

Separable Differential Equations

A **separable** differential equation can be written as:

$$\frac{dy}{dx} = f(x)g(y).$$

(or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or $\frac{dy}{dx} = \frac{g(y)}{f(x)}$.)

Example: Find the explicit solution to

$$\frac{dy}{dx} = -3xy$$

with $y(0) = 4$.

You do: Find the explicit solution to

$$\frac{dy}{dx} = 2xy^2$$

with $y(2) = \frac{1}{5}$.

What if the initial condition was $y(2) = 0$?

Observations:

A 1st order differential equation can have:

1. No Solution

2. Infinitely many solutions (one "parameter" or "free constant", initial conditions not given)

3. A unique solutions (initial conditions given)

In a class on the theory of differential equations you would talk about this is more detail (conditions on the differential equations in order for a solution to exist and be unique). Read 2.4 and ask me questions if you are interested in learning more, but that is not required.

Example: Find an *implicit* solution to

$$\frac{dy}{dx} = \frac{3x + 1}{5y^4 - y}$$

with $y(2) = 1$.

Example: Find the general *explicit* solution to

$$2\frac{dy}{dx} = 3x^2(y^2 - 1)$$

Example:

A town currently has 2100 people

- The birth/death rate is proportional to the population size with a relative growth rate of $k = 0.03$.
- In addition, 100 people/year are immigrating into the city from elsewhere.

Let $P(t)$ be the number of people in the city in t years from now.

Find $P(t)$.